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The effect of the discrete nature of rain on weather radar signals

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Abstract. The effects of the discrete nature of rain on a radar signal are examined through numerical and experimental analysis. A numerical model was developed and applied to examine the effect of various sizes and aspect ratios of the radar measurement volume, and high-resolution Doppler radar data were compared to this. Through statistical signal analysis, the discrete nature of rain was seen to be most important for very small radar measurement volumes, as expected. The signal statistics of the measured signal were different from the simulated signal statistics, probably because of the spatial homogeneity assumed in the numerical model. An attempt to make use of the discrete nature of rain showed some possibilities, which need more research.

1 Introduction

As (research) radars gain spatial resolution, the signal will be affected more and more by the discrete nature of rainfall. To investigate this effect, a numerical radar signal simulator was written. This numerical model will be described in Sect. 2. Experimental data of a high resolution research radar, described in Sect. 3, will be analyzed and compared to the numerically simulated signals. The discrete nature of rainfall can complicate analysis of the radar signal, and introduce error. Statistical analyses will be done in Sect. 4 to determine the extent of these effects. However, new types of analysis also become possible when the discrete nature of rain becomes important. In Sect. 5, an attempt will be made to take advantage of this. Finally, conclusions are drawn in Sect. 6.

2 Numerical model

A numerical radar signal simulator was developed to analyze, among others, the effects of the discrete nature of rain on a radar signal. The model generates a radar signal by adding the individual contributions of many individual raindrops

$$V_r = \sum_{i=1}^{N} D_i^3 w(r_i, \theta) \mathrm{e}^{j \frac{4\pi}{\lambda} r_i}. \tag{1}$$

D_i (mm) and r_i (m) are the diameter and the distance from the radar, respectively, of the i^{th} raindrop, and λ (m) is the radar wavelength. The sum is over all raindrops in the 'rain volume', and $w(r_i, \theta)$ is a range and angle dependent measurement volume bounding function. The raindrops are advected through the rain volume using

$$r_i(t + \Delta t) = r_i(t) - v_i \Delta t. \tag{2}$$

The terminal velocity v (m s^{-1}) vs. diameter relation is a combination of two different relations that are valid in different diameter ranges. The $v - D$ relation for small diameters is given by Atlas and Ulbrich (1977)

$$v_a(D) = \alpha D^{\beta}, \tag{3}$$

with $\alpha = 3.778\,\mathrm{m\,mm^{-\beta}\,s^{-1}}$ and $\beta = 0.67$. The velocity-diameter relation that is used for large drops is (Atlas et al., 1973)

$$v_b(D) = a - b\mathrm{e}^{-cD}, \tag{4}$$

with $a = 9.65\,\mathrm{m\,s^{-1}}$, $b = 10.3\,\mathrm{m\,s^{-1}}$ and $c = 0.6\,\mathrm{mm^{-1}}$. The velocity-diameter relation that is used in the model (and in the rest of this paper) is a combination of the two

$$v(D) = \begin{cases} v_a & D \le D_a \\ \frac{D_b - D}{D_b - D_a} v_a + \frac{D - D_a}{D_b - D_a} v_b & D_a < D \le D_b \\ v_b & D > D_b \end{cases} \tag{5}$$

where D_a (≈ 0.78 mm) and D_b (≈ 3.08 mm) are the two solutions to $v_a = v_b$.

The raindrops that are used for generating the radar signal have diameters that are drawn from a truncated exponential distribution

$$f_{D_V}(D) = C_1 \mathrm{e}^{-\Lambda D} \qquad D_{min} \le D < D_{max}, \tag{6}$$

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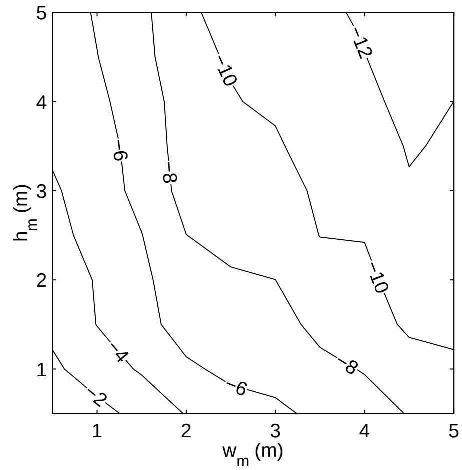

Fig. 1. Ratio of fluctuation powers of Z_D and $Z_r - Z_D$ as a function of the measurement volume width (w_m) and height (h_m). Contour levels indicate $10 \cdot {}^{10}\log(F)$.

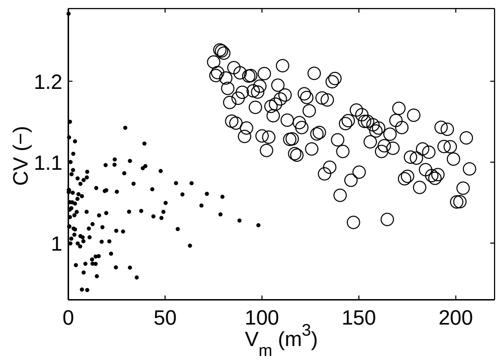

Fig. 2. Coefficients of variation of Z_r for the simulated signal (dots) and for the measured radar signal (circles) as a function of the measurement volume.

and zero elsewhere. C_1 is a normalization constant and Λ (mm^{-1}) given by (Marshall and Palmer, 1948)

$$\Lambda = 4.1 R^{-0.21}, \tag{7}$$

where R ($\mathrm{mm\,h}^{-1}$) is the rainfall intensity. The raindrop arrival process is a Poisson process, and the spatial distribution is homogeneous.

3 Experimental setup

TARA (Transportable Atmospheric RAdar) is a high-resolution Polarimetric Doppler FM-CW research radar (see Heijnen et al., 2002). It operates at a central frequency of 3.2975 GHz, has a beam width of 2.2° and has a near-field limit of $r = 200$ m. Its maximum sweep repetition frequency is 1000 Hz, so that after range-processing it gives a complex value of V_r for each range cell every 10^{-3} s. The frequency excursion used was sawtooth-shaped and had an amplitude of 52 MHz, which leads to a range resolution of $\Delta r = 2.885$ m. The range processing was done using a fast Fourier transform with a Hamming window to suppress sidelobes.

On September 22, 2002, TARA, located at Cabauw, The Netherlands, recorded raw data in a vertically pointing mode, while only one polarization was used. A rain event passed Cabauw shortly after 10:00 UTC, with intensities of up to $50\,\mathrm{mm\,h}^{-1}$ (recorded by disdrometers on the ground). A small portion of this data (around the time of maximum rainfall intensity) will be used throughout this paper.

4 Statistical analysis

Signal statistics can be used to determine the extent to which the discrete nature of rain plays a role in the radar signal.

At a given range, the radar reflectivity factor as measured by a radar, Z_r ($\mathrm{mm}^6\,\mathrm{m}^{-3}$), can be written as

$$Z_r = \frac{C_r \cdot |E|^2}{V_m} = \frac{C_r E E^*}{V_m}, \tag{8}$$

where C_r ($\mathrm{mm}^6\,\mathrm{m}^2\,\mathrm{V}^{-2}$) is a radar constant, V_m (m^3) is the measurement volume and E ($\mathrm{V\,m}^{-1}$) is the electric field received by the radar

$$E = \frac{1}{C_r^{1/2}} \sum_{i=1}^{N} w(r_i, \theta) D_i^3 \mathrm{e}^{j\frac{4\pi}{\lambda} r_i} = \frac{V_r}{C_r^{1/2}}. \tag{9}$$

The radar reflectivity factor Z_D ($\mathrm{mm}^6\,\mathrm{m}^{-3}$) is often defined as (Battan, 1973)

$$Z_D = \frac{1}{V_m} \sum_{i=1}^{N} D_i^6, \tag{10}$$

where V_m (m^3) is the size of the measurement volume. However, it is usually calculated from the incoherent radar signal, which, using Eqs. (8) and (9), and assuming a rectangular measurement volume bounding function, can be written as

$$Z_r = Z_D + \frac{1}{V_m} \sum_{i=1}^{N-1} \sum_{k=i+1}^{N} D_i^3 D_k^3 \cos\left(\frac{4\pi}{\lambda}(r_i - r_k)\right). \tag{11}$$

It can readily be seen that the expectation value of the second term on the right-hand side of Eq. (11) is zero if the raindrops are homogeneously distributed in the measurement volume.

Radar signals were simulated as described in section 2, using a rainfall intensity of $R = 30\,\mathrm{mm\,h}^{-1}$ and rectangular measurement bounding functions in both the range and the angular direction. The same rain was used to generate 100 5-second radar signals for cylindrical measurement volumes ranging from 0.5 m to 5.0 m in diameter (w_m) and from 0.5 m to 5.0 m in height (h_m), all in 0.5 m increments. The ratio of the signal fluctuations caused by the two components (see Eq. 11) of the incoherent radar signal are a measure of the importance of the discrete nature of the rain. This ratio can be expressed as

$$F=\sqrt{\frac{\left\langle(Z_D-\langle Z_D\rangle)^2\right\rangle}{\left\langle(Z_r-Z_D-\langle Z_r-Z_D\rangle)^2\right\rangle}}, \tag{12}$$

and is plotted as a function of the measurement volume scale in Fig. 1. The figure shows that as the measurement volume becomes smaller, the fluctuation power due to the discrete nature of rain becomes more and more important (F closer to 1). Whether the volume increases due to its height or due to its width does not seem to influence the the ratio of fluctuation powers.

Marshall and Hitschfeld (1953) have shown that for spatially uniformly distributed scatterers, in the limit of an infinite expected number of particles, the radar signal is distributed exponentially. One property of the exponential distribution is that its mean and standard deviation are equal. So, the coefficient of variation

$$CV=\frac{\sqrt{\left\langle(Z_r-\langle Z_r\rangle)^2\right\rangle}}{\langle Z_r\rangle} \tag{13}$$

for uniformly distributed drops should be equal to 1. Any deviation from 1 would indicate either that the drops are not distributed uniformly, or that there are simply not enough drops in the measurement volume. Fig. 2 shows the coefficient of variation as a function of the measurement volume for the simulated signal and for the measured signal. The measurement volumes of the measured signal are varied by taking the signal at different ranges, so that any variation with measurement volume may also be related to variation of precipitation with range. For the simulated signals, CV deviates most from 1 at very small volumes, indicating the larger effect of the discrete nature of rain. For the measured signal, the deviation of CV from 1 is most likely due to the fact that the drops are not uniformly distributed.

5 Doppler spectrum analysis

When investigating Doppler spectra for radars with small measurement volumes, signatures of individual drops may be seen. The extent to which this is feasible depends on the size and aspect ratio of the measurement volume, the rainfall intensity and the time over which the Doppler spectrum is taken. At a given time, the number of raindrops in a given diameter class per unit volume is

$$\rho_{V,c}=\rho_V\int_{D_l}^{D_u} f_{D_V}(D)\mathrm{d}D, \tag{14}$$

where ρ_V (m^{-3}) is the total volumetric drop density (see Uijlenhoet and Stricker, 1999) and D_l and D_u (both mm) are the lower and upper values of the diameter class, respectively. The total volumetric drop density is given by

$$\rho_V=\frac{R}{6\pi\cdot 10^{-4}\int_{-\infty}^{\infty} D^3 v(D) f_{D_V}(D)\mathrm{d}D}. \tag{15}$$

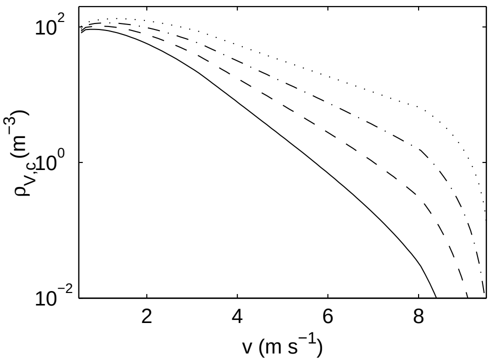

Fig. 3. Volumetric drop densities per velocity interval as a function of the Doppler velocity for different rainfall intensities. Solid line: $R=3\,\mathrm{mm\,h^{-1}}$; dashed line: $R=10\,\mathrm{mm\,h^{-1}}$; dash-dotted line: $R=30\,\mathrm{mm\,h^{-1}}$; dotted line: $R=100\,\mathrm{mm\,h^{-1}}$.

Using Eq. (6), the expression for the volumetric drop density per velocity class can be written as

$$\rho_{V,c}=\frac{R\int_{D(v_l)}^{D(v_u)} \mathrm{e}^{-\Lambda D}\mathrm{d}D}{6\pi\cdot 10^{-4}\int_{D_{min}}^{D_{max}} D^3 v(D)\mathrm{e}^{-\Lambda D}\mathrm{d}D}. \tag{16}$$

Another quantity that needs to be examined is the number of drops that fall through a surface in space per unit time per diameter class

$$\rho_{A,c}=\rho_A\int_{D_l}^{D_u} f_{D_A}(D)\mathrm{d}D, \tag{17}$$

where $f_{D_A}(D)$ (mm^{-1}) is the probability density function for drops falling through a surface per unit time (see again Uijlenhoet and Stricker, 1999)

$$f_{D_A}(D)=v(D)f_{D_V}(D). \tag{18}$$

The total number of drops falling through a surface per unit time is

$$\rho_A=\frac{R}{6\pi\cdot 10^{-4}\int_{-\infty}^{\infty} D^3 f_{D_A}(D)\mathrm{d}D}. \tag{19}$$

Per class, this becomes

$$\rho_{A,c}=\frac{R\int_{D(v_l)}^{D(v_u)} v(D)\mathrm{e}^{-\Lambda D}\mathrm{d}D}{6\pi\cdot 10^{-4}\int_{D_{min}}^{D_{max}} D^3 v(D)\mathrm{e}^{-\Lambda D}\mathrm{d}D}. \tag{20}$$

These drop densities per velocity class need to be solved numerically because of the inversion of the $v(D)$ relation (Eq. 5). The results of this for class widths of $0.1\,\mathrm{m\,s^{-1}}$ (the class widths obtained using standard TARA processing), $D_{min}=0.05\,\mathrm{mm}$ and $D_{max}=8.0\,\mathrm{mm}$ are shown in Figs. 3 and 4. These graphs show that, in theory, it should be possible to recognize individual drops from a time series of a

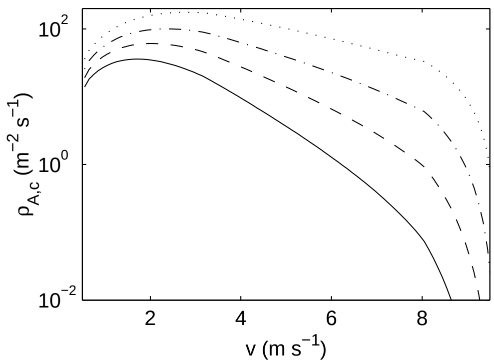

Fig. 4. Drop densities at a surface per velocity interval as a function of the Doppler velocity for different rainfall intensities. Solid line: $R = 3\,\mathrm{mm\,h^{-1}}$; dashed line: $R = 10\,\mathrm{mm\,h^{-1}}$; dash-dotted line: $R = 30\,\mathrm{mm\,h^{-1}}$; dotted line: $R = 100\,\mathrm{mm\,h^{-1}}$.

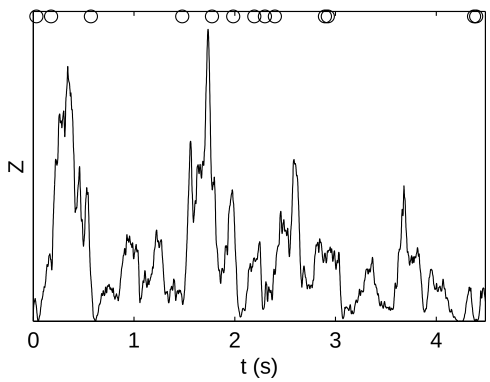

Fig. 5. Time series of the $9.0\,\mathrm{m\,s^{-1}}$ velocity bin of the Doppler spectrum for the simulated signal for $w_m = 0.5\,\mathrm{m}$ and $h_m = 0.5\,\mathrm{m}$. Circles indicate the entry of a drop into the measurement volume.

single velocity bin of the Doppler spectrum. For this, it is necessary that the rainfall intensity is not too high and that the measurement volume is small.

Finding the signatures of individual drops in the Doppler spectrum can be done by realizing that the Doppler spectrum for one drop is the convolution of the spectrum that would result from an unbounded measurement volume and the spectrum of the range bounding function. The finite extent of the range bounding function will cause some spectral leakage, which can be reduced by using windowing functions in range- and Doppler Fourier transforms. An example of a time series of the $v = 9.0\,\mathrm{m\,s^{-1}}$ velocity class Doppler spectrum of the simulated signal for the smallest measurement volume is given in Fig. 5. The Doppler spectrum was calculated using a fast Fourier transform with a 0.512 s rectangular time window, which was then advanced over the signal. The circles in Fig. 5 indicate the entry of a drop with a velocity that is in the given class into the measurement volume.

It can be seen from the figure that some of the drops can actually be recognized in the graph. However, because of spectral leakage and other noise, this graph cannot be directly used to obtain the entry times and the number of all the drops in the particular velocity bin.

6 Conclusions

The discrete nature of rain becomes increasingly important as the radar measurement volume decreases in size. The fluctuation power of the defined Z_D is about half that of the fluctuation power of $Z_r - Z_D$ for a volume of about $1\,\mathrm{m}^3$. When reaching volumes of actual radars, this difference has become at least an order of magnitude. The aspect ratio of the measurement volume does not have much effect on the signal fluctuations, which is to be expected while the measurement volume height is still much larger than one wavelength. For small volumes, the distribution of the radar signal, given a uniform spatial drop distribution, deviates from an exponential distribution. This deviation decreases with increasing volume. The deviation of the measured radar signal from the exponential distribution is much greater, possibly because of non-uniform spatial drop distributions.

For small volumes, it should be possible to see the signatures of large drops in the radar signal. In the large diameter range, there will be very little drops in the radar measurement volume at a time. A glance at the time series of a high-velocity bin of the Doppler spectrum of a simulated signal for a very small radar measurement volume shows that it is indeed possible to see the signatures of individual drops. However, more analyses need to be done before any definite conclusions can be drawn on the feasibility of this.

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