NONLINEARITIES AND FRACTAL DISTRIBUTIONS

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In analyses of phenomena leading to fractal distributions there prevail opinions that the cause of such distributions are critical phenomena in phase transitions or SOC models. The power distributions are also associated with deterministic chaos generated by low-dimensional nonlinear systems. The nonlinearity is considered as a necessary condition for scale invariance and fractal statistics. However, an explanation how and what kinds of nonlinearities lead to fractal distributions is lacking. In this contribution we investigate an influence of nonlinearity of the model onto its output behaviour. In many cases the relation between the "input" x and the "output" y of a system can be expressed by the nonlinear transformation $y = g(x)$. We investigate the kind of nonlinearity of the model which transforms an input random variable with Poisson distribution onto an output variable with long-tail distribution over a wide range of scales. It occurs that a very wide class of nonlinear increasing functions g leads to the power-like output distributions. Then, nonlinear relations of the type $y = g(x)$ can be solutions of random differential equations, or a system of random differential equations which describe some physical phenomena. Some kinds of random differential equations are analysed and discussed. The contribution shows in elementary way the causes of universality of fractal distributions in many branches of science.