SCALING PHENOMENA AND MODELLING IN FINANCE

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The classical model for the dynamics of financial time series, which goes back to Louis Bachelier, assumes the logarithmic prices to be a Brownian motion. One of the properties it does not appropriately account for is scaling behaviour. Already in 1963 Mandelbrot observed anomalous scaling in daily cotton prices. He suggested to model the price increments by Levy-distributed random variables. This would imply infinite variance for the price changes. Since then, anomalous scaling behaviour has been observed under different circumstances. Important examples are:

- the absolute values of logarithmic price changes (returns) as a function of the corresponding time intervals,
- moments of the absolute returns, and
- the extremal behaviour of the return distributions.

The observations typically point in the direction of multifractal properties. For adequate modelling of financial time series, volatility clustering must also be taken into account, but the variance should stay finite. Until the mid of the nineties the improvement of modelling in finance concentrated mainly on volatility clustering (ARCH and GARCH models) while neglecting nearly totally scaling behaviour. Only recently, with the more wide-spread use of high-frequency data, scaling behaviour has been modelled. Examples are on one hand the fractionally integrated GARCH model and the heterogenous ARCH model (HARCH) and on the other hand multifractal approaches as stochastic cascade models. HARCH and multifractal models assume time horizons extending from the order of minutes to the order of months. A particular interesting application of scaling behaviour using analogies from geophysics is the construction of a Scale of Market Shocks similar to the Richter scale for earthquakes.