

FLOW THROUGH ISOTROPIC MULTIFRACTAL POROUS MEDIA IN \mathbf{R}^D

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We make a renormalization analysis of flow through random porous media under the condition that the hydraulic conductivity $\mathbf{K}(\underline{\mathbf{x}})$ is an isotropic lognormal multifractal field in D -dimensional space, with log spectral density $S_{\ln \mathbf{K}}(\underline{\mathbf{k}}) \propto |\underline{\mathbf{k}}|^{-D}$. We find that the resulting hydraulic gradient $\nabla H(\underline{\mathbf{x}})$ and specific flow $\underline{\mathbf{q}}(\underline{\mathbf{x}})$ are also multifractal and that their renormalizations under space contraction involve both random scaling and random rotation. The multifractal parameters that describe the scaling properties of ∇H and $\underline{\mathbf{q}}$ are obtained analytically as functions of the space dimension D and the codimension parameter $C_{\mathbf{K}}$ of \mathbf{K} . We derive the spectral density tensors of ∇H and $\underline{\mathbf{q}}$ and compare our results with conventional perturbation theory. The fields ∇H and $\underline{\mathbf{q}}$ are anisotropic at large scales, but are locally isotropic. The scale of isotropy depends on $C_{\mathbf{K}}$ and mildly on D . While ∇H is a conservative field, $\underline{\mathbf{q}}$ is not; hence separate scaling results apply to the bare and partially dressed flows. We use the partially dressed results to find the effective permeability, which is identical to an expression conjectured by Matheron (1967). Two-dimensional simulations confirm the analytical results.